Factor of conformity in modular dynamic study of frame type constructions

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Abstract

The combined conducted analytical, numerical and experimental studies to specify the factor of conformity in modular dynamic tests of frame type constructions are presented in the current report. It is well known that experimental studies are carried out by creating scale models of the real systems based on linear, mass, inertia, elastic and other scales. Resulting characteristics, describing the dynamic behavior of the scale models, after multiplying by the corresponding modules provide information about the behavior of the real ones. The factors of conformity to the own circular frequency and the amplitude of the forced vibrations, caused by kinematic sinusoidal interference, are analytically deduced in the report. The factors are applicable to one bay, one story frame with rigid external and internal links. The theory of the Deflection method of the Structural mechanics is used in the analytical determination of the coefficients. The numerical verification of the accuracy of the coefficients is realized through the appropriate software for dynamic analysis of structures, working based on the finite element method. Approximate experimental verification of the deduced dependencies is done through dynamic analysis of two models by Stand for modular dynamic tests of structures, subjected on seismic impacts. Developed methodology for determining the coefficients of conformity can be used for other class structures, including those with limited or infinite number of degrees of freedom.

Keywords: scale dynamic model; numerical experiments; testbed experiments; factor of conformity.

1. Introduction

The dynamic study of the contemporary building structures is a complicated and complex task. The solution of the task is held in order analytical, numerical and finally experimental study. The analytical study is applicable to simpler structures or to separate their details. For more complex structures analytical part of the study is limited to preparatory work for the next stages. Numerical study is related to the decision of the analytical deduced mathematically records, describing the dynamics of the system. The holding of the numerical study requires the development of programs or simulation models by means of software packages for computational mathematics as Matlab, Maple, Mathematica. Many freedom of the research and practically unlimited possibilities for numerical

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dynamic analysis provide the new software packages based on FEM (SAP 2000, RSAP, Ansys, Comsol etc.). The third and final stage of the full dynamic study is the experimental. Various scale models of the real structures are created for conducting of the experimental research. The scale models are subjected to relevant impacts to determine their dynamic behavior. Information about the dynamic behavior of the real structures is obtained by processing of the characteristics, describing the dynamics of the scale models. This involves the use of respectively factors of conformity, giving connection between the characteristics of the both models.

A complex dynamic study of one bay, one story frame with rigid external and internal links is conducted in this report. The main part of the study is devoted to determining of the factor of conformity to the floor displacement, caused by sinusoidal kinematic impacts in the supporting nodes. The amplitude of the force undamped vibrations of the system is analyzed to determine the above mentioned factor. The value of the factor for such a simple system gives the range, in which the latter should be sought, in more complex systems and more complicated undeterminate impact. The simple character of the test system allows analytical, numerical and experimental solution of the problem.

**Nomenclature**

\[ \begin{align*}
\text{m, } m_1 & \quad \text{mass of the storey of the real and the scale model} \\
\text{EI, } EI_1 & \quad \text{stiffness coefficient of the column of the real and the scale model} \\
\text{l, } l_1 & \quad \text{height of the column of the real and the scale model} \\
\text{\omega, } \omega_1 & \quad \text{own circular frequency of the real and the scale model} \\
\theta & \quad \text{force frequency} \\
\mu = \theta / \omega & \quad \text{relative frequency} \\
A, A_1 & \quad \text{amplitude of story vibrations of the real and the scale model} \\
\rho_l, \rho_m & \quad \text{linear and mass factor} \\
\rho_{EI} & \quad \text{stiffness factor} \\
\rho_{\omega} & \quad \text{factor of the own circular frequency} \\
\rho_A & \quad \text{factor of the amplitude of story vibrations}
\end{align*} \]

2. **Analytical, numerical and experimental procedure**

Two similar dynamic models are used for the conduct off combined determination of the factor of conformity. The models are shown in Fig. 1. As characteristics for the first (Fig. 1. (a)), can be used standard average statistical for such a frame values. The second is a scale model with random mass, stiffness and linear scales. The stiffness of the beam in both models is accepted for infinity. The axial distance between the columns is not fixed, because it does not influence on the dynamic behavior of the systems.

![Fig. 1. (a) A typical 2D frame; (b) Scale model of 2D frame.](image-url)
2.1. Analytical study

The analytical study is based on the theoretical formulations of the discipline of Structural Mechanics. [k] form Deflection method is used in study of the dynamic behaviour of the systems. The sequence of application of the method is as follows. The moment diagram from a single displacement of the applied linear support is defined initially (Fig. 2. (a)). Next step is the determination of the same diagram from the amplitude of the kinematic effect on the supporting nodes (Fig. 2. (b)). As a kinematic effect is accepted sinusoidal vibration of the supporting nodes with forced frequency in the range of 0.75-0.85 from the base one.

The diagrams, caused by the single displacement and amplitude value of the forced sinusoidal effects for the scale models of Fig. 1. (b) are with the same shape as those of Fig. 2. The values of the last depends on the characteristics of scale models.

The next step is determination of the reactions of the applied linear support. The reaction from a single displacement of the linear support is used for determination of the own circular frequency of the system. The reaction from the amplitude value of the sinusoidal impact is used to determine the displacement from that impact.

![Fig. 2. (a) Moment diagram from v₁=1; (b) Moment diagram from the amplitude value of the sinusoidal impact.](image)

The reaction from a single displacement (Fig. 2. (a)) is defined as the amount from the shear forces in columns and has the form

\[
k_{11} = \frac{2 \times l^2 \times EI}{l^3}
\]

The own frequency of vibration of the model from the same figure is calculated by the formula for frequency of the vibrating system with 1 degree of freedom (DOF).

\[
\omega = \sqrt{\frac{k_{11}}{m}} = \sqrt{\frac{24 \times EI}{l^3 \times m}}
\]

The own frequency of vibration of the scale model can be calculated by a similar formula.

\[
\omega_j = \sqrt{\frac{24 \times EI_j}{l_j^3 \times m_j}}
\]

The module (factor of conformity) of the own frequencies is determined as the ratio of the last.

\[
\rho_\omega = \frac{\omega}{\omega_j} = \sqrt{\frac{24 \times EI \times l_j^3 \times m_j}{l^3 \times m \times 24 \times EI_j}} = \sqrt{\frac{\rho_{EI} \times l_j^3 \times m_j}{\rho_{EI} \times l^3 \times m}}
\]
The reaction from the amplitude value of the sinusoidal impact in supporting nodes is proportional of the reaction from a single displacement.

\[ K_{x_0} = -\frac{2 \times 12 \times EI}{l^3} \cdot \ddot{x}_0 = -k_{11} \cdot \ddot{x}_0 = -\omega^2 \cdot m \cdot \ddot{x}_0 \]  

(5)

The displacement of the floor level from this impact will be determined by the condition of dynamic equilibrium in direction of the displacement.

\[ (k_{11} - \theta^2 \cdot m) \cdot v_{x_0} + K_{x_0} = 0 \]  

(6)

\[ (\omega^2 \cdot m - \theta^2 \cdot m) \cdot v_{x_0} - \omega^2 \cdot m \cdot \ddot{x}_0 = 0 \]  

(7)

\[ v_{x_0} = \frac{\omega^2 \cdot m}{(\omega^2 - \theta^2) \cdot m} \cdot \ddot{x}_0 = \frac{1}{(I - \mu^2)} \cdot \ddot{x}_0 \]  

(8)

From the theory of the free undamped vibrations of a system with 1 DOF is known, that the law of motion will be the sum of the solutions of the homogeneous differential equation and private integral. The amplitude of the private integral in this type of vibration is equal to the displacement of the floor level from amplitude value of the impact.

\[ v = C_1 \cdot \cos(\omega \cdot t) + C_2 \cdot \sin(\omega \cdot t) + v_{x_0} \cdot \sin(\theta \cdot t) \]  

(9)

At zero initial conditions the coefficient \( C_1 \) is zero and the coefficient \( C_2 \) will have the type

\[ C_2 = -\frac{\theta}{\omega} \cdot v_{x_0} = -\mu \cdot v_{x_0} \]  

(10)

The final form of the law of motion will be

\[ v = -\mu \cdot v_{x_0} \cdot \sin(\omega \cdot t) + v_{x_0} \cdot \sin(\theta \cdot t) \]  

(11)

A short analysis of the vibrations, described by above law shows the following. The amplitude of the total vibrations will vary within the range \( v_{\text{max}} - v_{\text{min}} \). The maximum value of the total amplitude will be in simultaneously close to the unit value and equal sign of the amplitudes of the two component sine functions. When forced frequency of the impact is less than the own frequency of the system, amplitude will look like
The increase of the amplitude to a maximum value will occur twice for the time for which vibration by their own sinewave become one more than the vibration under the forced. The period of change of the amplitude can be found using the formula

\[ T_v = \frac{2 \pi}{\omega_v - \theta} \]  

(13)

The general appearance of the graph of the summary vibrations of the system can be viewed on Fig. 3. Finally, it can be found the factor of conformity for maximum amplitude in two large-scale model

\[ \rho_v = \frac{A_{\text{max}}}{A_{i_{\text{max}}}} = \frac{\xi_{i0} \cdot I - \mu_i}{\xi_{0} \cdot I - \mu} \]  

(14)

Usually the ratio of the amplitudes of the forced kinematic impact is equal to the linear module. Then the formula (14) will get the type

\[ \rho_v = \rho_i \cdot \frac{I - \mu_i}{I - \mu} \]  

(15)

2.2. Numerical studies

The numerical study is realized in two areas. The first is the area of software system Matlab / Simulink. Program for visualization of the described by formula (11) law of forced vibrations has been developed in Matlab environment. The vibrations in standard and scale model of the frame of Fig. 1 are visualized by the program. The maximum amplitudes in the both vibrations are determined by the graphs. The ratio of the maximum amplitude gives the factor of conformity for the floor displacements. The factor of conformity is compared with that, obtained by formula (14). It can be used formula (15), if the ratio of amplitudes of kinematics impact is equal to the linear module.

Graphs of the vibrations in random data for the both models are shown in Fig. 4. The data, at which the graphs are derived, are given below.

![Fig. 4. (a) Graph of total vibration of the system; (b) Graph of total vibration of the scale model.](image)

The factor of conformity, obtained as ratio of the maximum amplitudes of the force vibrations, is 2.523. The factor of conformity, analytically derived on the base of formula (15), is 2.5. There are quite close values, which numerically verify the correctness of the analytical procedure.
Numerical analysis is also realized in the area of the software system Ansys. A series of numerical experiments with real and scale models were conducted. The value of the factor of conformity varies in the range of 2-8% of the theoretical coefficient, computed by the formulas (14) и (15). Due to the limited format of the report, results from these studies are not shown.

2.3. Experimental study

Stand for modular dynamic tests of frame type structures [6] is used for experimental study. The stand is a part of the facilities of the Laboratory for numerical and experimental dynamic modeling at the UACEG, Sofia, Bulgaria (www.dlab-uacg-bg.eu).

Two models, with scale linear and mass characteristics, are tested on the stand. Photos of the two models are shown in Fig. 5. The characteristics of both models are given on the right of the photos.

\begin{align*}
\text{Fig. 5. (a) Experimental study of standard model; (b) Experimental study of scale model.}
\end{align*}

Based on the shown in the figure characteristics, the natural frequencies of the both models are derived by formulas (2) and (3). Then, through the system for controlling the movement of the stand, the models are driven by the specified sinusoidal impact. The amplitudes of the impact are selected so, that their ratio to be equal to the linear scale of the models, which is 1.275. The forced frequencies are selected so, that the relative frequency to be equal respectively to 0.8 for the first and 0.75 for the second model. The movement of the floor surface is recorded by a special system [5], then the results are processed in the area of Matlab program. The results from the movement of the two models are visualized in Fig. 6.

\begin{align*}
\text{Fig. 6. (a) Graph of story vibration of the model 1; (b) Graph of story vibration of the scale model.}
\end{align*}

The factor of conformity, with respect to the maximum displacement, obtained as ratio of the first to the second gives 1.468. By formula (15) the same factor is obtained 1.594. The difference of about 9% is within the range of permissible under experimental studies.
3. Discussions

The close results in terms of the factor of conformity, from the conducted combined dynamic analysis of plane one bay, one story frames, verify the correctness of the methodology. It should be borne in mind, that the results are most accurate at the relative forced frequency in the range 0.65-0.9. In this interval of variation of the relative frequency, the graph indicates several internal vibrations within the period calculated in formula (13). At closer values of the own and the forced frequency approaches the resonance mode, which is dangerous, especially in experimental tests. At a relative frequency less than 0.65 within the period \( T_v \), it is observed a small number of vibrations, it being possible the maximum amplitude of the vibration to be significantly different from that, calculated by the formula (12).

In drawing up the scale models, most important is the linear scale. The latter should not be more than 4-5, because in the formula for frequency module is of a third degree. For larger values of the linear scale will be obtained systems with different in order natural frequencies, which will result in a large difference in the results with respect to the displacements.

In models with larger linear scales, can vary by stiffness and mass scale, so that the factor of conformity to the own frequencies to be in the range 1.5 – 2.5.

4. Conclusion

The proposed combined methodology for determining a factor of conformity for the displacements of the story plates of a standard and the scale models, is an open system and can be changed. It can be applied in plane frames with 1-3 joint internal or supporting links. With such a change would be modified formulas (2) and (3) for the own frequencies, but will not change the next formulas, describing the factors of conformity, including and that for the own frequency.

The methodology, especially in its numerical and experimental section, can be applied to study the dynamic behavior of multi storey or multi bay frames. The recommendations, given above for the range of variability of the relative forced frequency, should refer to the ratio of forced frequency to the first own frequency of the plane systems.

With respect to the impacts, the methodology assumes easily change of the deterministic with undeterministic interferences, for example recorded by vibrometer seismic interferences. The average amplitude of the impacts should be the average amplitude of the actual impact, multiplied by the linear scale of the numerical or the experimental models.

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References


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